Marker: Ja hazels
AT 4016
8062

Continuing from last day...

$$J = \begin{bmatrix} -a_1S_1 - a_2S_{12} & -a_2S_{12} \\ a_1C_1 + a_2C_{12} & a_2C_{12} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \underbrace{\begin{bmatrix} -a_1S_1 - a_2S_{12} & -a_2S_{12} \\ a_1C_1 + a_2C_{12} & a_2C_{12} \end{bmatrix}}_{J_{xy}} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

If $\dot{x}$ and $\dot{y}$ are given, then we can find

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} J^{-1} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$$

When J_{xy} is non-singular. $\det(J_{xy}) \neq 0$

$$\det(J_{xy}) = a_1 a_2 \sin\theta_2$$

$\therefore$ there is a singular solution when

$$\sin\theta_2 = 0$$

$$\theta_2 = 0, \frac{\pi}{2}k \qquad k \in \mathbb{Z}$$

Note: This occurs when the planar robot's arm is fully extended, hence it shows that it is very important to study the jacobian, and how it works.

Singularity Decoupling.

Here we use the spherical wrist to help with decoupling.

For manipulators having spherical wrist it is possible to split the problem of singularities into 2 separate problems.

- Computation of arm singularities resulting from the first 3 or more links.

- Computation of the wrist singularities resulting from the motion of the wrist joints.

EX: Consider a 6 DOF manipulator (3DOF arm, 3 DOF wrist)

The jacobian is:

$$J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

3×3 matrices.

Since the spherical wrist joints are all revolute:

$$\begin{bmatrix} J_{12} \\ J_{22} \end{bmatrix} = \begin{bmatrix} \hat{z}_3 \times (O_6 - O_3) & \hat{z}_4 \times (O_6 - O_4) & \hat{z}_5 \times (O_5 - O_6) \\ \hat{z}_3 & \hat{z}_4 & \hat{z}_5 \end{bmatrix}$$

Since $\hat{z}_3$, $\hat{z}_4$, $\hat{z}_5$, $\hat{z}_6$ intersect at the wrist center

$$O_6 = O_5 = O_4 = O_3$$

Then

$$J_{12} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

And the overall jacobian

$$J = \begin{bmatrix} J_{11} & 0 \\ J_{21} & J_{22} \end{bmatrix}$$

arm singularities

wrist singularities

Then

$$\det(J) = \det(J_{11}) \cdot \det(J_{22})$$

Note: The singularities of the spherical wrist are standardised, so most of the time it is not nessisary.

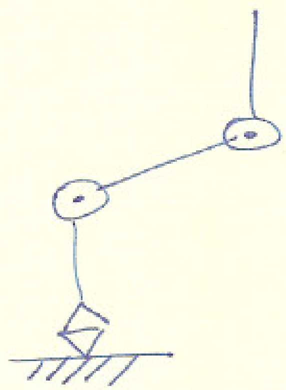
Wrist Singularities.

$$\det(J_{22}) = 0$$

$$\boxed{\begin{aligned} \theta_5 &= 0 \\ \theta_5 &= \pi \end{aligned}}$$ full extention

Arm Singularities.

EX:



$J_{11} = ?$

J_{21} ... who cares.

$$J_{11} = \begin{bmatrix} -S_1(a_2C_2 + a_3C_{23}) & -C_1(a_2S_2 + a_3S_{23}) & -a_3C_1S_{23} \\ C_1(a_2C_2 + a_3C_{23}) & -S_1(a_2S_2 + a_3S_{23}) & -a_3S_1S_{23} \\ 0 & a_2C_2 + a_3C_{23} & a_3C_{23} \end{bmatrix}$$

$$\det(J_{11}) = 0$$

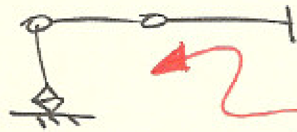
$$-a_2 a_3 S_3 (a_2 C_2 + a_3 C_{23}) = 0$$

$$S_3 = 0$$

$$a_2 C_2 + a_3 C_{23} = 0$$

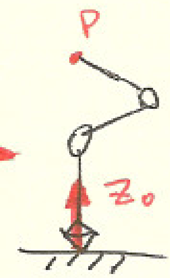
Hence there are singularities when.

$$\theta_3 = 0, \pi k \qquad k \in \mathbb{Z}$$



link 2 and link 3 are in line.

$$a_2 C_2 + a_3 C_{23} = 0$$



there is a singularity when P intersects z_0

If the jacobian is not square then a solution for $\cancel{\dot{q}}$

$$\underset{6\times 1}{\dot{x}} = \underset{6\times n}{J(q)}\, \underset{n\times 1}{\dot{q}}$$

exists if the rank of the augmented matrix is equal to the rank of $J(q)$

$$\text{rank}\left(J(q)\right) = \text{rank}\left(\underbrace{\left[J(q), \dot{x}\right]}_{\text{augmented matrix.}}\right)$$

Note: Rank is equal to the largest none zero square in the matrix. ?

Redundant Manipulators

(more than 6 DOF)

$$\underbrace{\dot{x}}_{6\times 1} = \begin{bmatrix} v \\ \omega \end{bmatrix} = \underbrace{J}_{6\times n} \underbrace{\dot{q}}_{n\times 1}$$

So the problem here is quite simple, find $\dot{q}$ satisfying the preceeding equation

A varible solution is to formulate the problem given v and ω find $\dot{q}$ satisfying the preceeding equation and minimise the quadratic cost function of the joint velocities.

$$g(\dot{q}) = \frac{1}{2}\dot{q}^T W \dot{q}$$

Where $W = W^T$ is a positive definate matrix

This in turn minimises the overall kinetic energy.

EX:

$$\dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \qquad W = \begin{bmatrix} 100 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\dot{q}^T W \dot{q} = 100\,\dot{q}_1^2 + \dot{q}_2^2$$

Here it is clear that $\dot{q}_1$ is weighted much more, it will be much smaller, there will be much less velocity.